# Alignment Sensitivities in ILC Damping Rings

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**Abstract**

We compare quadrupole and sextupole alignment sensitivities in two proposed designs for the ILC damping rings with alignment sensitivities in the KEK-ATF prototype damping ring. The comparisons are based on simple analytical estimates, and do not take into account tuning procedures.

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## 1 Introduction

The ILC damping rings will need to achieve normalized vertical emittance below 2 $\mu$m. At an operating energy of 5 GeV, this corresponds to a geometric emittance of around 2 pm. The lowest vertical emittance achieved in any storage ring to date is a little under 5 pm, at the KEK-ATF [1]. Vertical emittance is generated by vertical dispersion and betatron coupling, which in turn arise from magnet misalignments. Generally, vertical emittances of a few pm require highly precise initial alignment of the magnets, followed by beam-based alignment techniques and effective coupling compensation using, for example, skew quadrupoles. The complex behavior of the vertical emittance makes it difficult to quantify the level of difficulty of achieving a specified vertical emittance in any given ring. However, it is possible to make simple analytical estimates that state, for example, the response of the vertical emittance to vertical movement of the sextupoles. It is to be expected that the more sensitive the emittance to magnet motion, the more difficult it will be to achieve a low emittance.

In this note, we compare the emittance sensitivity of two proposed lattice designs for the ILC damping rings with the emittance sensitivity of the KEK-ATF. Although the ILC damping ring designs are very different, the KEK-ATF provides a useful benchmark because of the experience of tuning the lattice for very low vertical emittance. A more thorough understanding of the vertical emittance in the proposed damping ring designs will require simulations including orbit and coupling correction systems.

We first present the formulae we shall use to compare the sensitivities of the different lattices to various magnet misalignments. We then give the relevant parameters for the different lattices, and a comparison of the sensitivities. Note that the results depend on details of the lattice designs (including tunes and beta functions). Since the designs are not fixed, these results should be considered as only indicative of the behavior of the different lattices, and not definitive.

## 2 Sensitivity Formulae

The sensitivity of the orbit, dispersion and coupling in a lattice to magnet motion depends on the magnet strengths and lattice functions. It is convenient to define the following quantities:

$$\Sigma_{1\mathrm{O}} = \sum_{\text{quadrupoles}} \beta_y (k_1 L)^2 \tag{1}$$

$$\Sigma_{1\mathrm{D}} = \sum_{\text{quadrupoles}} \beta_y \eta_x^2 (k_1 L)^2 \tag{2}$$

$$\Sigma_{1\mathrm{C}} = \sum_{\text{quadrupoles}} \beta_x \beta_y (k_1 L)^2 \tag{3}$$

$$\Sigma_{2\mathrm{D}} = \sum_{\text{sextupoles}} \beta_y \eta_x^2 (k_2 L)^2 \tag{4}$$

$$\Sigma_{2\mathrm{C}} = \sum_{\text{sextupoles}} \beta_x \beta_y (k_2 L)^2 \tag{5}$$

The numeric subscript on the $\Sigma_{**}$ indicates whether the summation is performed over the quadrupoles or the sextupoles, and the alphabetic subscript identifies the quantity as relevant for the orbit, dispersion, or betatron coupling. The $k_1 L$ are the integrated normalized quadrupole strengths, and the $k_2 L$ are the integrated normalized sextupole strengths.

In terms of the above quantities, we can write the following approximate relationships:

$$\frac{\langle y^2 \rangle}{\langle \sigma_y^2 \rangle} \simeq \frac{\langle \Delta Y_q^2 \rangle}{8 \epsilon_y \sin^2 \pi \nu_y} \Sigma_{1\mathrm{O}} \tag{6}$$

$$\frac{\epsilon_y}{\langle \Delta \Theta_q^2 \rangle} \simeq \frac{J_x}{J_y} \frac{1 - \cos 2\pi\nu_x \cos 2\pi\nu_y}{(\cos 2\pi\nu_x - \cos 2\pi\nu_y)^2} \epsilon_x \Sigma_{1\mathrm{C}} + J_\epsilon \frac{\sigma_\delta^2}{\sin^2 \pi\nu_y} \Sigma_{1\mathrm{D}} \tag{7}$$

$$\frac{\epsilon_y}{\langle \Delta Y_s^2 \rangle} \simeq \frac{J_x}{J_y} \frac{1 - \cos 2\pi\nu_x \cos 2\pi\nu_y}{4\,(\cos 2\pi\nu_x - \cos 2\pi\nu_y)^2} \epsilon_x \Sigma_{2\mathrm{C}} + J_\epsilon \frac{\sigma_\delta^2}{4 \sin^2 \pi\nu_y} \Sigma_{2\mathrm{D}} \tag{8}$$

Here, $\langle y^2 \rangle$ is the mean square vertical orbit distortion; $\langle \Delta Y_q^2 \rangle$ is the mean square vertical quadrupole misalignment; $\langle \Delta Y_s^2 \rangle$ is the mean square vertical sextupole misalignment; $\langle \Delta \Theta_q^2 \rangle$ is the mean square quadrupole rotation about the beam axis; $J_x$, $J_y$ and $J_\epsilon$ are the damping partition numbers; $\nu_x$ and $\nu_y$ are the betatron tunes, and $\sigma_\delta$ is the rms natural energy spread. These expressions assume that the misalignments are random and uncorrelated, that the betatron coupling is dominated by the lowest-order difference resonance, and that the dispersion in the dipoles and wigglers is not correlated. These assumptions are not necessarily valid for the damping rings. In particular, when calculating the contribution of the vertical dispersion to the emittance, it can be important to consider the dispersion in the wiggler separately from the rest of the lattice. This is because the radiation from the wiggler typically dominates over the radiation from the dipoles. This emphasizes the need to study the emittance tuning by detailed simulations; however, bearing these issues in mind, we can proceed to estimate the sensitivities of the different lattices.

We define the following three measures of the lattice sensitivity:

**Quadrupole jitter sensitivity** is the rms quadrupole misalignment that will generate an orbit distortion equal to the beam size for a specified emittance. This is found from (6).

**Quadrupole rotation sensitivity** is the rms quadrupole rotation that will generate a specified vertical emittance. This is found from (7).

**Sextupole alignment sensitivity** is the rms sextupole vertical misalignment that will generate a specified vertical emittance. This is found from (8).

## 3 Sensitivity Comparisons

Table 1 gives the relevant lattice parameters for the lattices that we compare. The vertical emittance given in the table is the achieved value for the ATF, and the specified operating value for the "TESLA Dogbone" [2] and "ILC Small" [3] damping rings. Table 2 gives the results of the sensitivity calculations. Note that smaller values indicate a greater sensitivity to the corresponding alignment.

Table 1: Lattice parameters.

| | ATF | TESLA Dogbone | ILC Small |
|---|---|---|---|
| Circumference [m] | 139 | 17000 | 6114 |
| Energy [GeV] | 1.28 | 5.0 | 5.066 |
| Horizontal Emittance [nm] | 1.0 | 5.1 | 5.5 |
| Vertical Emittance [pm] | 5.0 | 1.4 | 1.4 |
| Energy Spread [$10^{-3}$] | 0.55 | 1.3 | 1.5 |
| Horizontal Damping Partition | 1.6 | 1.0 | 1.0 |
| Vertical Damping Partition | 1.0 | 1.0 | 1.0 |
| Horizontal Tune | 15.141 | 76.310 | 56.584 |
| Vertical Tune | 8.759 | 41.180 | 41.618 |

Table 2: Lattice sensitivities.

| | ATF | TESLA Dogbone | ILC Small |
|---|---|---|---|
| Quadrupole Jitter [nm] | 241 | 80.7 | 198 |
| Quadrupole Rotation [$\mu$rad] | 825 | 40.5 | 58.3 |
| Sextupole Alignment [$\mu$m] | 45.6 | 11.3 | 40.4 |

We see that both the TESLA Dogbone and the ILC Small damping rings are more sensitive than the ATF to magnet misalignments, although the specified emittance might be easier to achieve in the ILC Small damping ring. We should emphasize that the sensitivities should *not* be interpreted as survey alignment tolerances, since achieving the specified emittances will require correction of the dipsersion and betatron coupling, which we have not considered in these simple estimates. Detailed simulation studies will be needed to specify the requirements on initial alignment, and the performance of the diagnostics and coupling correction systems.

## References

[1] Y. Honda et al, "Achievement of Ultralow Emittance Beam in the Accelerator Test Facility Damping Ring", Phys. Rev. Lett. **92**, 054802 (2004).

[2] TESLA Technical Design Report, Chapter 5, DESY 2001-011, 2001. http://tesla.desy.de/new_pages/TDR_CD/PartII/chapter05/chapter05.pdf

[3] A. Xiao, "An Alternative TESLA Damping Ring Lattice Design at Fermilab", 2004. http://awolski.lbl.gov/ILCDR/Documentation.htm